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Filtering of the output signal of dynamically tuned gravimeters

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Abstract— In this paper, we present the gyroscopic gravimeter state algorithm, with digital processing of sensor orientation information. We researched errors in gyroscopic gravimeter state assessment and typical errors influences on the gyroscopic gravimeter law of motion. The algorithm allows efficient filtration of most random and systematic errors.

Index Terms— filtering, gravimeter, gravimetric measuring complex, Cramer's rule, estimation algorithm, precession oscillation period, nonlinear distortion

1 INTRODUCTION

THE need to improve the accuracy and speed of a double-ring dynamically tuned gravimeter in gravimetric measuring complex (GMC) with automatic processing of information is caused by the need to establish effective and easy-to-implement algorithms of assessment of the double-ring dynamically tuned gravimeter (DG). Gravimeters which are the primary sensing element of gravimetric measuring complex should have high metrological characteristics: accuracy, sensitivity, speed and reliability [1]. The raising level of these requirements encourages to conduct research to improve the gravimeters accuracy and speed.

Analysis of research. In recent years, there appeared many research papers on the development and research of optimal and suboptimal modifications of algorithms for discrete signal of unbalanced gyroscope filtering. Review of the literature and practical work on the aviation gravimetry [1-4] showed that during the study of dynamically tuned gravimeters, the impact of gravimetric errors caused by nonlinear distortions of gyroscope trajectory were not taken into account: precession oscillations damping through viscous type torques action on the sensor element; non-synchronization of precession oscillations; the discrepancy between the value of the angular precession vibrations frequency used in the estimation algorithms, and the value of the angular precession oscillation frequency of the sensing element; interferences that distort the sensing element mode of motion. However, the impact of these errors, if not taken into account, can be prohibitively large (on the order of 10 - 30 mGal). Therefore, the goal is to improve the accuracy and speed of measurement of double-ring dynamically tuned gravimeter in gravimetric measuring complex by removing these errors. However, the literature gives no information on the impact of these errors on the accuracy and speed of the gravimeter [1-8].

The aim of this paper is to solve development problems of the errors assessment theory of the dynamically tuned gravimeter with digital information processing.

The main part.

Let's consider the state of development of the algorithm of DG digital processing of information in the northward orientation of the sensing element (SE): analytically examine the error estimates due to the inadequacy of the accepted original model and the actual signal of DG, and errors due to kinematic nonlinearities.

Movement of SE that observed with the angle sensor (AS) function can be represented as

$$\alpha(t)=R^{V}+\alpha_1(t)+\varepsilon(t), \tag{1}$$

where R^{V} - angle between AS zero and calculated north direction; $\alpha_1(t)$ - the current state of the SE angle relative to the North, which is determined by solving the equation

$$\ddot{\alpha}_1+2\xi_1\dot{\alpha}_1+\omega_0\sin\alpha_1=0 \tag{2}$$

where ω_0 - circular precession frequency of small oscillations of SE; $\varepsilon(t)$ - precession motion trajectory of SE curvature; ξ_1 - damping parameter.

In the case of small oscillations of SE ($\sin\alpha_1\approx\alpha_1$) the function $\alpha_1(t)$ can be represented in the form

$$\alpha(t)=Ae^{-\xi_1 t}\sin(\omega t+\varphi), \tag{3}$$

where $\omega=\sqrt{\omega_0^2-\xi_1^2}$; $A;\varphi$ - amplitude and initial phase precession of oscillations respectively.

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Based on the (3) SE model of motion expression, observed by AS, can be written as

$$\alpha(t) = \hat{R}^V + \hat{A}e^{-\xi_1 t}\sin(\omega t + \hat{\varphi}), \tag{4}$$

where $\hat{R}^V$ - calculated angle between AS zero and calculated area to the north; $\hat{A}, \hat{\varphi}$ - calculated A, φ.

In general, in expression (4) $\hat{R}^V, \hat{A}, \hat{\varphi}, \omega, \xi_1$ are unknown values.

Taking the assumptions into account, motion model (4) can be described by the expression

$$\alpha(t) = \hat{R}^V + \hat{A}_c \sin\omega t + \hat{A}_s \cos\omega t, \tag{5}$$

where $\hat{A}_c = \hat{A}\cos\hat{\varphi}, \hat{A}_s = \hat{A}\sin\hat{\varphi}$.

State vector to be estimated in this case, can be written as

$$\hat{x}_N = \left[\hat{R}^V \quad \hat{A}_c \quad \hat{A}_s\right].$$

For examining the problem of estimation by least squares:

$$F_N = \sum_{i=1}^{n}\left(\hat{R}^N + \hat{A}_c \sin\omega t_i + \hat{A}_{s\cos}\omega t_i - \alpha_i\right)^2. \tag{6}$$

Here $\alpha_i = \alpha(t_i)$ SE angular position at time $t_i = (1-i)\Delta t \ \ (t = 1, n)$ when monitoring movement of SE in observation time T_c; Δt - discrete of time interval of information obtaining. $n = T_c \Delta t^{-1} + 1$ - the number of observed samples for T_c.

Function minimum F_N is reached at

$$\frac{\partial F_N}{\partial \hat{R}^V} = \frac{\partial F_N}{\partial \hat{A}_c} = \frac{\partial F_N}{\partial \hat{A}_s} = 0. \tag{7}$$

Conditions (7) are equivalent to the matrix equation

$$c^N \hat{x}_N = z_N. \tag{8}$$

$$c^N = \begin{bmatrix} n & \sum_{i=1}^{n}\sin\omega_0 t_i & \sum_{i=1}^{n}\cos\omega_0 t_i \\ \sum_{i=1}^{n}\sin\omega_0 t_i & \sum_{i=1}^{n}\sin^2\omega_0 t_i & \sum_{i=1}^{n}\sin\omega_0 t_i \cos\omega_{10} t_i \\ \sum_{i=1}^{n}\cos\omega_0 t_i & \sum_{i=1}^{n}\sin\omega_0 t_i\cos\omega_{10} t_i & \sum_{i=1}^{n}\cos^2\omega_0 t_i \end{bmatrix},$$

$$z_N = \left[\sum_{i=1}^{n}\alpha_i \quad \sum_{i=1}^{n}\alpha_i\sin\omega_0 t_i \quad \sum_{i=1}^{n}\alpha_i\cos\omega_0 t_i\right]$$

Let us define the observation system (8). It is known that homogeneous system of algebraic equations has a unique solution when its main determinant is not zero. We show under certain conditions $\det c^N \neq 0$.

Within matrix theory it is known that the Gramm's determinant is a presentation

$$D = \begin{bmatrix} (x_1 x_1) & (x_1 x_2) & \ldots & (x_1 x_m) \\ (x_2 x_1) & (x_2 x_2) & \ldots & (x_2 x_m) \\ (x_m x_1) & (x_m x_2) & \ldots & (x_m x_m) \end{bmatrix}. \tag{9}$$

where $\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_m$ - a set of n- dimensional vectors, $(x_i x_j)$ - scalar product of vectors $\overline{x}_i, \overline{x}_j$, and $i, j \in [1, n]$. Qualifier D - positive when vectors $\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_m$ are linearly independent. If we accept

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}, \quad x_2 = \begin{bmatrix} \sin\omega t_1 \\ \sin\omega t_2 \\ \cdot \\ \cdot \\ \cdot \\ \sin\omega t_n \end{bmatrix}, \quad x_3 = \begin{bmatrix} \cos\omega t_1 \\ \cos\omega t_2 \\ \cdot \\ \cdot \\ \cdot \\ \cos\omega t_n \end{bmatrix}. \tag{10}$$

the determinant of the system (8) is the Gramm's determinant. So if $\overline{x}_1, \overline{x}_2, \overline{x}_3$ are linearly independent, $\det c^N > 0$ and the system (8) is always solvable in only one way. We find conditions where $\overline{x}_1, \overline{x}_2, \overline{x}_3$ are linearly independent.

Consider the case when $n = 3$. Linear dependence of vectors $\overline{x}_1, \overline{x}_2, \overline{x}_3$ is equivalent to the system of equations

$$b_1\overline{x}_1 + b_2\overline{x}_2 + b_3\overline{x}_3 = 0, \tag{11}$$

$$D' = \begin{bmatrix} 1 & 0 & 1 \\ 1 & \sin\lambda & \cos\lambda \\ 1 & \sin 2\lambda & \cos 2\lambda \end{bmatrix} = 2\sin\lambda\cos\lambda. \tag{12}$$

where $\lambda = \omega\Delta t$.

It follows that

$$\lambda \neq \pi k; \quad \lambda \neq \pi\left(\frac{1}{2} + 2k\right); \quad (k = 0, \pm 1, \pm 2). \tag{13}$$

$D' \neq 0$ i.e. vectors $\overline{x}_1, \overline{x}_2, \overline{x}_3$ are linearly independent. Obviously, if the conditions (13) vectors $\overline{x}_1, \overline{x}_2, \overline{x}_3$ are linearly independent and at $n > 3$.

Thus, if the conditions

$$n \geq 3; \quad \Delta t \neq \frac{\pi k}{\omega}; \quad \Delta t \neq \frac{\pi}{2\omega} + \frac{2\pi k}{\omega}, \tag{14}$$

system (8) can be solved in only one way. We prove the solvability condition of the system (8) is equivalent to examining evidence of observation system.

Let us solve the system (8), after adjusting the amount of trigonometric functions to their final expression. After replacement, we write the system (8) in the form

$$c^N_{\lambda,n}\hat{x}_N = z^N_{\lambda,n}, \tag{15}$$

where

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$$c_{\lambda,n}^{N}=\begin{bmatrix} n\sin\frac{\lambda}{2} & \sin\frac{n\lambda}{2}\sin\frac{n-1}{2}\lambda & \sin n\lambda\cos\frac{(n-1)}{2}\lambda \\ 4\sin\frac{n\lambda}{2}\sin\frac{n-1}{2}\lambda\cos\frac{\lambda}{2} & n\sin\lambda-\sin n\lambda\cos(n-1)\lambda & \sin n\lambda\sin(n-1)\lambda \\ 4\sin\frac{n\lambda}{2}\cos\frac{n-1}{2}\lambda\cos\frac{\lambda}{2} & \sin n\lambda\sin(n-1)\lambda & n\sin\lambda+\sin n\lambda\cos(n-1)\lambda \end{bmatrix},$$

$$z_{\lambda,n}^{N}=\begin{bmatrix} z_1^N & z_2^N & z_3^N \end{bmatrix}^T=$$

$$=\left[\sin\frac{\lambda}{2}\sum_{i=1}^{n}\alpha_i\, 2\sin\lambda\sum_{i=1}^{n}\alpha_i\sin\lambda(i-1) 2\sin\lambda\sum_{i=1}^{n}\alpha_i\cos\lambda(i-1)\right]^T.$$

The system (15) solution can be offered by Gauss sampling main element or Cramer's rule. In this method to solve the system elements vector $\hat{x}_N$ can be represented as

$$\begin{aligned}\hat{R}^N&=\left(\det c_{\lambda,n}^N\right)^{-1}\left(A_{11}^N z_1^N+A_{21}^N z_2^N+A_{31}^N z_3^N\right);\\ \hat{A}_c&=\left(\det c_{\lambda,n}^N\right)^{-1}\left(A_{12}^N z_1^N+A_{22}^N z_2^N+A_{32}^N z_3^N\right);\\ \hat{A}_s&=\left(\det c_{\lambda,n}^N\right)^{-1}\left(A_{13}^N z_1^N+A_{23}^N z_2^N+A_{33}^N z_3^N\right),\end{aligned} \quad (16)$$

where A_{ij}^N - algebraic additional elements c_{ij}^N matrix $c_{\lambda,n}^N$.

Expanding the expression $\hat{R}^N$, we obtain

$$\hat{R}^N=\left[nk_1-4\cos\frac{\lambda}{2}\sin^2\frac{n\lambda}{2}\right]^{-1}\left(k_1S_1^N+k_2S_2^N+k_3S_3^N\right), \quad (17)$$

where

$$k_1=\sin\frac{\lambda}{2}\left(n\sin\lambda+\sin n\lambda\right),$$

$$k_2=-2\sin\frac{n\lambda}{2}\sin\frac{n-1}{2}\lambda\sin\lambda,$$

$$k_3=-2\sin\frac{n\lambda}{2}\cos\frac{n-1}{2}\lambda\sin\lambda,$$

$$S_1^N=\sum_{i=1}^{n}\alpha_i;\quad S_2^N=\sum_{i=1}^{n}\alpha_i\sin\lambda(i-1);\quad S_3^N=\sum_{i=1}^{n}\alpha_i\cos\lambda(i-1)$$

Ratings of amplitude A and the initial phase φ precession oscillations are determined through A_c and A_s and expressions

$$\hat{A}=\sqrt{\hat{A}_c^2+\hat{A}_s^2}, \quad (18)$$

$$\hat{\varphi}=\begin{cases}\arcsin\frac{A_s}{A_c}, & A_c\geq 0;\\ \pi-\arcsin\frac{A_s}{A_c}, & A_c\leq 0.\end{cases}$$

Let us study the error of assessment of the DG. For this, we present the solution of the differential equation (2) with $\sin\alpha\approx\alpha-\frac{\alpha^3}{3!}$ in the form of

$$\alpha_1(t)=A_0e^{-\xi_1 t}\sin(pt+\varphi_0)+A_1e^{-\xi_1 t}\sin 3(pt+\varphi_0), \quad (19)$$

where $p\cong\omega_0\left(1-\frac{A_0^2}{16}\right)$, $A_1\cong\frac{A_0^3}{192}$, $\omega_0>>\xi_1$.

Expanding $\alpha_1(t)$ in Taylor series in the parameters p and ξ in a point of $(\omega_0,0)$ and, leaving only two terms of the expansion, we obtain

$$\begin{aligned}\alpha_1(t)&\approx A_{0C}\sin\omega_0 t+A_{0S}\cos\omega_0 t+\\ &+\frac{A_0^3}{192}\sin 3(\omega_0 t+\varphi_0)-\omega_0 t\frac{A_0^3}{192}\cos 3(\omega_0 t+\varphi_0)-\\ &-\xi_1 tA_0\sin(\omega_0 t+\varphi_0)\end{aligned} \quad (20)$$

On the other hand, Taylor series decomposition models feature motion of SE (5) the parameters $\hat{R}^N,\hat{A}_c,A_s,\omega$ in a point of $\left(\hat{R}_0{}^N,\hat{A}_{0C},A_{0S},\omega_0\right)$ and leaving the first two terms of the expansion in a row, we obtain

$$\begin{aligned}\alpha(t_i)&=\hat{R}_0^N+\Delta\hat{R}^N+\hat{A}_{0C}\sin\omega_0 t+\\ &+\hat{A}_{0S}\cos\omega_0 t+\Delta\hat{A}_C\sin\omega_0 t_i+\Delta\hat{A}_{0S}\cos\omega_0 t_i+\\ &+\Delta\omega t_1\hat{A}_0\cos(\omega_0 t+\hat{\varphi}_0)+\varepsilon(t_i)\end{aligned}. \quad (21)$$

where $\Delta\hat{R}^N=\hat{R}^N-\hat{R}_0^N$, $\Delta\hat{A}_C=\hat{A}_C-\hat{A}_{0C}$, $\Delta\hat{A}_S=\hat{A}_S-\hat{A}_{0S}$, $\Delta\omega=\omega-\omega_0$ - assessment of the error.

After substituting expressions (20) and (21) in the function (6), we obtain

$$F_N=\sum_{i=1}^{n}\left(\Delta\hat{R}^N\Delta\hat{A}_C\sin\omega_0 t_i+\Delta\hat{A}_{0S}\cos\omega_0 t_i-\alpha_0^N(t_i)\right), \quad (22)$$

$$\begin{aligned}\alpha_0^N(t_i)&=-A_0\left(\Delta\omega+\omega_0\frac{A_0^2}{16}\right)t_i\cos(\omega_0 t+\varphi_0)-\\ &-A_0\xi t\sin(\omega_0 t+\varphi_0)+\frac{A_0^3}{192}\sin 3(\omega_0 t_i+\varphi_0)+\varepsilon(t)\end{aligned}. \quad (23)$$

Conditions to achieve a minimum functional equivalent of matrix equation

$$c_{\lambda 0,n}^N\hat{\delta}^N=f^N, \quad (24)$$

where the matrix $c_{\lambda 0,n}^N\hat{\delta}^N=\hat{f}^N$ $\lambda=\lambda_0$; $\lambda_0=\omega_0\Delta t$;

$$\hat{\delta}^N=\begin{bmatrix}\Delta\hat{R}^N & \Delta\hat{A}_C & \Delta\hat{A}_S\end{bmatrix}^T;$$

$$f^N=\left[\sin\frac{\lambda_p}{2}\sum_{i=1}^{n}\lambda_N(t_i)2\sin\lambda_0\sum_{i=1}^{n}\alpha_N^0(t_i)\sin\lambda_0(i-1)\cdot 2\sin\lambda_0\sum_{i=1}^{n}\alpha_N^0(t_i)\cos\lambda_0(i-1)\right]^T$$

Using Cramer's rule for finding $\Delta\hat{R}^N$ we obtain

$$\begin{aligned}\Delta\hat{R}^N&=\left(nk_1^0-4\cos\frac{\lambda_0}{2}\sin^2\frac{n\lambda_0}{2}\right)^{-1}\times\\ &\times\left(k_1^0f_1^N+k_2^0f_2^N+k_3^0f_3^N\right)\end{aligned}, \quad (25)$$

where $k_j^0=k_j\Big|_{\lambda=\lambda_0}$, $\left(j=\overline{1,3}\right)$

$$f_1^N=\sum_{i=1}^{n}\alpha_N^0(t_i),\qquad f_2^N=\sum_{i=1}^{n}\alpha_N^0(t_i)\sin\lambda_0(i-1),$$

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$$f_3^N = \sum_{i=1}^{n} \alpha_N^0(t_i)\cos\lambda_0(i-1).$$

Expanding expression (25) and releasing further for convenience when writing code λ we obtain an evaluation error expression

$$\Delta\hat{R}^N = d_1(\lambda,n)\times$$
$$\times A_0\left[\left(\frac{A_0^2}{16}+\frac{\Delta\omega}{\omega_0}\right)\sin\left(\varphi_0+\frac{n-1}{2}\lambda\right)-\frac{\xi}{\omega_0}\cos\left(\varphi_0+\frac{n-1}{2}\lambda\right)\right]- , \quad (26)$$
$$-d_2(\lambda,n)\frac{A_0^3}{192}\sin 3\left(\varphi_0+\frac{n-1}{2}\lambda\right)+d_3(\lambda,n,\varepsilon(t_i))$$

where

$$d_1(\lambda,n)=\frac{\lambda}{2\sin\lambda}\cdot\frac{\sin\frac{n\lambda}{2}\left(\sin\lambda+\frac{n}{2}\sin 2\lambda\right)-(n\sin\lambda)^2\cos\frac{n\lambda}{2}}{n\sin\frac{\lambda}{2}(n\sin\lambda+\sin n\lambda)-4\cos\frac{\lambda}{2}\sin^2\frac{n\lambda}{2}}$$

$$d_2(\lambda,n)=\frac{(1+2\cos\lambda)(\cos\lambda+\sin n\lambda)\sin\frac{n\lambda}{2}\sin n\lambda-\cos\lambda\sin^3\frac{n\lambda}{2}(n\sin\lambda+\sin n\lambda)}{\cos\lambda(1+2\cos\lambda)\left[n\sin\frac{\lambda}{2}(n\sin\lambda+\sin n\lambda)-4\cos\frac{\lambda}{2}\sin^2\frac{n\lambda}{2}\right]}$$

$d_3[\lambda,n,\varepsilon(t_i)]$ - errors due to distortion of the observed law of SE motion.

At $\Delta t \le 0{,}01T_0$ $\left(T_0 = 2\pi\omega_0^{-1}\right)$ we appropriately simplify the expression (26) with $\Delta t \to 0$, $T_H = const$ making the transition to the limit $\Delta\hat{R}^N$, thus we obtain

$$\Delta\hat{R}^N = \sum_{i=1}^{n}\Delta R^N, \quad (27)$$

$$\Delta\hat{R}_1^N = -d_2(u)\frac{A_0^3}{192}\sin 3\left(\varphi_0+\frac{u}{2}\right),$$

$$\Delta\hat{R}_2^N = -d_1(u)\frac{\xi}{\omega_0}A_0\cos\left(\varphi_0+\frac{u}{2}\right),$$

$$\Delta\hat{R}_3^N = -d_1(u)\frac{A_0^3}{16}\sin\left(\varphi_0+\frac{u}{2}\right),$$

$$\Delta\hat{R}_4^N = -d_1(u)\frac{\Delta\omega}{\omega_0}A_0\sin\left(\varphi_0+\frac{u}{2}\right),$$

$$\Delta\hat{R}_5^N = \lim_{\substack{\Delta t\to 0\\ T_C=const}} d_3[\lambda,n,\varepsilon(t_i)] = d_3[u,\varepsilon(t_i)],$$

$$d_1(u)=\frac{(u+\sin u)\sin\frac{u}{2}-u^2\cos\frac{u}{2}}{u(u+\sin u)-8\sin^2\frac{u}{2}},$$

$$d_2(u)=\frac{2}{3}\,\frac{3\cos\frac{u}{2}\sin^2 u-\sin\frac{3}{2}u(u+\sin u)}{u(u+\sin u)-8\sin^2\frac{u}{2}},$$

$u = \omega_0 T_C$.

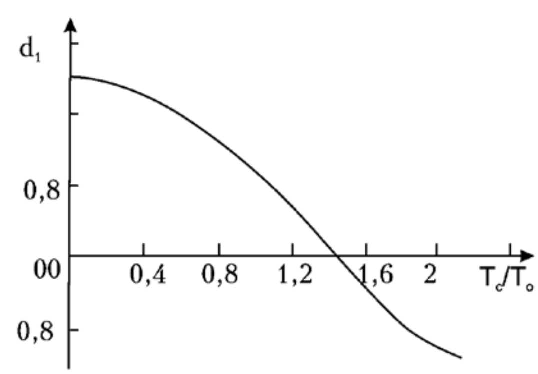
$\Delta\hat{R}_1^N$ - assessment of the error component caused by the third harmonic of the SE law of motion; $\Delta\hat{R}_2^N$ - component estimation error caused by equivalent friction; $\Delta\hat{R}_3^N$ - Component estimation error caused by non-isochronism fluctuations; $\Delta\hat{R}_4^N$ - component estimation error caused by inaccurate consideration of frequency oscillations precession; $\Delta\hat{R}_5^N$ - component estimation error caused by distortion of the law of motion as a result of wrong moments on SE and errors in the AS.

Let us analyze the components $\Delta\hat{R}_j^N$ $\left(j=\overline{1,4}\right)$ errors of assessment $\Delta\hat{\hat{R}}^N$. As can be seen from their expressions maximums defined parameters $A_0, \xi\omega_0^{-1}, \Delta\omega\cdot\omega_0^{-1}$ and time-dependent observation information that is proportional coefficients $d_1(\omega)$ and $d_2(\omega)$.

As can be seen from the graph (Fig. 1a) $d_1(T_H)$ dependence monotonically decreases to $1{,}465T_0$ then changes sign. The points at which $d_1(T_H)=0$ are optimal in the sense of becoming a zero maximum values of the components $\Delta\hat{R}_2^N$, $\Delta\hat{R}_3^N$, $\Delta\hat{R}_4^N$ and observation time. The smallest optimal observation time is $T_C = 1{,}465T_0$.

Coefficient $d_2(T_C)$ (Fig. 1b) also decreases monotonically $T_C = 0{,}640T_0$ then changes sign and performs damped oscillations repeatedly when crossing the x-axis. Some zero $d_2(T_C)$ can be found directly

$$T_C = kT_0,\ (k=1,2,3). \quad (28)$$



a)

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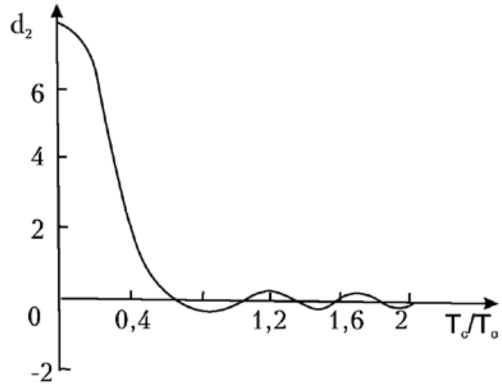

b)

Fig. 1. Dependence of the impact d_1, d_2 errors $\Delta\hat{R}_1^N$ from T_0

Coefficient $d_1(T_C)$ and $d_2(T_C)$ can be considered as some gain of the error $\Delta\hat{R}_j^N$. If measurement $d_1(T_C)$ can be called smooth the whole strength of obtaining information gain no more than twice, the coefficient $d_2(T_C)$ has sharp drops, causing 6 - 8 time gain of error estimates in $T_C \le 0{,}2T_0$ and more than fourfold reduction when $T_C > 0{,}6T_0$.

Fig. 2 shows the dependence of the maximum values of the components of error estimation $\max\left(\Delta\hat{R}_i^N\right)$:

- error $\Delta\hat{R}_1^N$ (Figure 2, curve 1) seen in precession amplitude fluctuations $A_0 = 10^\circ$ and $T_C < 0{,}6T_0$; at $T_C > 0{,}6T_0$ the maximum error of less than 2 angle;
- error $\Delta\hat{R}_1^N$ (Fig. 7.2, curve 2) at $T_C = (0{,}2...0{,}3)T_0$ may exceed 100 angle at $A = 10^\circ$;
- whereas, in actual use $\Delta\omega \cdot \omega_0^{-1} \le 5 \cdot 10^{-4}$ and $\xi_1 \cdot \omega_0^{-1} \le 10^{-4}$ at $A_0 = 1^\circ$ and $T_C \ge 0{,}1T_0$ we obtain $\max\left(\Delta\hat{R}_2^N\right) < 1$ angle and $\max\left(\Delta\hat{R}_4^N\right) \le 3{,}5$ angle (Figure 2, curve 3).

On the other hand, the expression of error evaluation shows that the error components are harmonic functions of initial phase fluctuations φ_0 with a period 2π and $2 \cdot 3^{-1}\pi$. For a given observation time information for each of the components of the error $\Delta\hat{R}_j^N$ is optimal, in the sense of transformation in this part zero initial phase, namely:

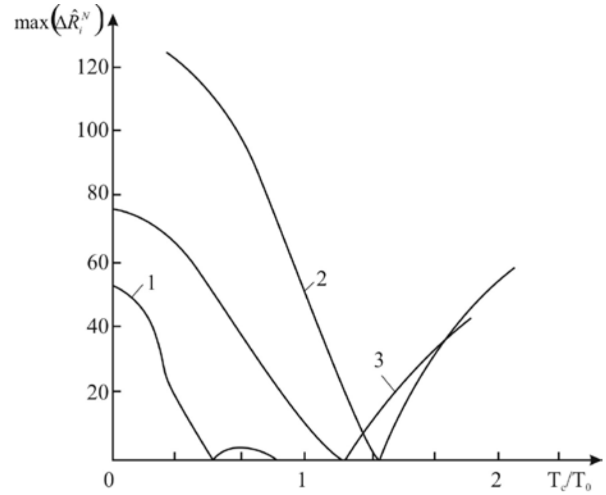

Fig. 2. Dependence $\left(\Delta\hat{R}_i^N\right)$ from $T_C : 1 - i = 1; 2 - i = 3; 3 - i = 2; 4$

We turn to the analysis of component error assessment of $\Delta\hat{R}_5^N$, due to distortions law of motion as a result of unwanted moments on the SE and the noise in the AS control channel . According to expression (26) and with (25), we obtain

$$\bar{d}_3[\lambda, n, \varepsilon(t)] = \frac{k_0^1 \sum_{i=1}^{n} \varepsilon(t_i) + k_0^2 \sum_{i=1}^{n} \varepsilon(t_i) \sin\lambda(i-1) k_0^3 \sum_{i=1}^{n} \varepsilon(t_i) \cos\lambda(i-1)}{nk_0^1 - 4\cos\frac{\lambda}{2}\sin^2\frac{n\lambda}{2}}.$$

Through this passage to the limit in terms $\Delta t \to 0$ and $T_C = const$ we obtain an expression for the errors of assessment in the form:

$$(u + \sin u)\int_0^{T_C} \varepsilon(t)dt = 4\sin^2\frac{u}{2}\int_0^{T_C} \varepsilon(t)\sin\omega t dt - 2\sin u \int_0^{T_C} \varepsilon(t)\cos\omega t dt,$$

$$\Delta\hat{R}_5^N = \omega_0, \quad (29)$$

$$u(u + \sin u) = 8\sin^2\frac{u}{2}.$$

The influence of typical interference with gravimeter law of motion.

1. In the event of adverse moment at the SE, which varies linearly, interference $\varepsilon(t)$ has the form

$$\varepsilon(t) = k_\partial t, \quad (30)$$

where k_∂ - slope drift.

Substituting expression $\varepsilon(t)$ in the formula (29) gives

$$\Delta\hat{R}_{51}^N = \frac{1}{2}k_\partial T_C, \quad (31)$$

that is proportional to the slope of the drift error and observation time.

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2. In the event of SE on the date of exponential obstacle type $\varepsilon(t)$ in the form of

$$\varepsilon(t) = a_e e^{-\frac{\tau}{t}}, \tag{32}$$

where a_e - The value of obstacles to $t = 0$; τ - time of constant obstacles.

Expression (29) after substituting (32) and limiting transition $\Delta t \to 0$, $T_C = const$ has the form

$$-(u+\sin u)\left(e^{-\frac{u}{u_1}}-1\right) = \frac{2u_1}{u_1^2+1} \times$$

$$\times\left[2\sin^2\frac{u}{2}\left(e^{-\frac{u}{u_1}}-1\right) - u_1^{-1}\sin u\left(e^{-\frac{u}{u_1}}-1\right)\right]'$$

$$\Delta\hat{R}_{52}^N = a_Э u_1, \tag{33}$$

$$u(u+\sin u) = 8\sin^2\frac{u}{2}$$

where $u_1 = \omega\tau$.

As can be seen from the expression (33), the error $\Delta\hat{R}_{52}^N$ defined parameters of interference a_e, τ and depends on the observation data. The dependence of the error changes T_C presented in two charts at 1: $\tau = 100c$ and 2: $\tau = c$ and $a_t = 1$, thus the proposed error decreases monotonically with increasing observation time, and reduce errors more quickly than smaller time constant τ. If a limited time, observing this error can make a systematic error component evaluation.

3. Harmonic interference can be caused by unsteady thermal state of SE and the influence of periodic motions at a frequency of oscillation of the SE pendulum. In the first case, the interference with the period of change commensurate period of precession oscillations; the second - period interferences hundred times smaller than the oscillation period of precession SE. In both cases, the interference is given in the form

$$\varepsilon(t) = a_r\sin(\Omega t + \varphi_1) \tag{34}$$

here a_r - amplitude noise; Ω - angular frequency interference; φ_1 - phase shift between the interference and precession fluctuations.

After substitution of (34) into equation (29) and limiting transition $\Delta t \to 0$, $T_C = const$ we obtain an expression for error evaluation

$$\Delta\hat{R}_{53}^N = a_r d_{33}(\mu,u)\sin\left(\frac{\mu u}{2}+\varphi_1\right), \tag{34}$$

$$d_{33}(\mu,u) = \frac{\frac{2}{\mu}(u+\sin u)\sin\frac{\mu u}{2} - 4\sin\frac{u}{2}\left[\frac{\sin\frac{\mu+1}{2}u}{\mu+1}+\frac{\sin\frac{\mu-1}{2}u}{\mu-1}\right]}{u(u+\sin u)-8\sin^2\frac{u}{2}}, \tag{35}$$

$\mu = \dfrac{\Omega}{\omega_0}$, $u = \omega_0 T_H$.

Dependency errors (7) $\Delta\hat{R}_{53}^N$ and expression (35) infers, that the error $\Delta\hat{R}_{53}^N$ is harmonically damped, and the oscillation period of the sine wave is determined by the μ. Zero error value is determined from the expression

$$T_H = \frac{2T_0}{\mu}(k\pi - \varphi_1), \tag{36}$$

$(k = 1,2,3)$.

Increased frequency of change increases interference uncertainty for small observation intervals and a sharp decrease due to increased estimation error T_C. Thus, the $\mu = 100$ and $T_C \le 0{,}1T_0$ amplify intereference a hundred times, while $T_C \ge 0{,}15T_0$ reduces it a hundred times and at $T_C \ge 0{,}3T_0$ there is little effect on accuracy assessment.

Thus, the low-frequency noise estimation algorithm slightly choke at $0{,}1T \le T_C \le T_0$ the high-frequency noise $(\mu \ge 100)$ is effectively filtered at $T_C \ge 0{,}15T_0$, at $T_C = 0{,}2 - 0{,}3T_0$ there is virtually no errors assessment.

4. Analyze the estimation error due to the presence of the observed laws of motion noise meter control.

AS miss hindrance given in the form of white noise. To estimate the error on top use Holder's inequality

$$\Delta\hat{R}_{54}^N \le d_{34}(u)\Phi_\varepsilon(T_C), \tag{37}$$

where $d_{34}(u) = \sqrt{\dfrac{u(u+\sin u)}{u(u+\sin u)-8\sin^2\frac{u}{2}}}$,

$F_\varepsilon(T_C) = \sqrt{\dfrac{1}{T_C}\displaystyle\int_0^{T_C}\varepsilon^2(t)dt}$; where $F_\varepsilon(T_C)$ - Rms random function $\varepsilon(t)$ on the length of time $[0, T_C]$.

Dependence of the $d_{34}(u)$ from T_C showed that when $T_C \ge 0{,}2 \div 0{,}3T_0$ coefficient $d_{34}(u)$ is almost constant, therefore increasing $T_C \ge 0{,}2 \div 0{,}3T_0$ is inappropriate because the estimation error decreases significantly.

Conclusions :

Analysis of expression error evaluation showed that the assessment of the error has five elements, which are caused by: nonlinear distortion of the gravimeter sensing element trajectory, precession oscillations damping through viscous type torques action on the sensor element, non-synchronization of the precession oscillations, the discrepancy between the value of the angular precession vibrations frequency used in the estimation algorithms, and the value of the angular precession oscillation frequency of the sensing element, interferences that distort the sensing element mode of motion. The first two errors are directly proportional to the cube of precession oscillations amplitude, the second two errors are directly proportional to the first power of the precession fluctuations amplitude. For effective suppression of high-frequency noise and white noise barriers, the observation time should be 0.2 ... 0.3 of precession oscillation period.

Areas for further research:

1. Research and development of algorithm of assessment of the double-ring dynamically unstable equilibrium of custom gravimeter by the least squares method.

2. Comparative analysis of errors assessment by the method of the least squares and optimal Kalman filter.

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